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A note on asymptotic stability of an interval neutral delay–differential system☆

1. Introduction

The theory of neutral delay–differential systems is of both theoretical and practical interest. For a large class of electrical networks containing lossless transmission lines, the describing equations can be reduced to neutral delay–differential equations. Also, neutral systems often appear in the study of automatic control, population dynamics etc. During the past few years, the problem of stability analysis of neutral delay–differential systems has received great interest in the literature [1–3]. A number of stability criteria based on the characteristic equation approach, involving the determination of eigenvalues, measures and norms of matrices, have been presented [4,5].

Next, to state some known results, below we give a list of symbols used and their definitions.

$\mathcal{R}^n$ and $\mathcal{C}^n$	Sets of real and complex n-dimensional vectors
$\mathcal{R}^{n\times m}$ and $\mathcal{C}^{n\times m}$	Sets of $n \times m$ real and complex matrices
I_n and 0_n	$n \times n$ identity and zero matrices
$\mathcal{R}e(s)$	Real part of $s \in \mathcal{C}$
$\rho(A)$	Spectral radius of matrix A
$\lvert A\rvert$	Modulus matrix of matrix A
$\lVert A\rVert$	Spectral norm of matrix A; $\lVert A\rVert = \sqrt{\lambda_{\max}(A^*A)}$
$\mu(A)$	Matrix measure of matrix A; $\mu(A) = \frac{1}{2}\lambda_{\max}(A + A^*)$
$\bar{\sigma}(A)$	Largest singular value of matrix A
$\mathrm{diag}[a_1, \ldots, a_n]$	A diagonal matrix with a_i as its ith diagonal element
$A \le B$	The elements of A and B satisfy the inequality $a_{ij} \le b_{ij}$.

This work deals with the asymptotic stability of an interval neutral delay–differential system described by

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bx(t-\tau) + C\dot{x}(t-\tau), \\ &x(t) = \phi(t), \quad t \in [-\tau,\ 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, A, B and $C \in \mathcal{R}^{n\times n}$ are matrices whose elements vary in prescribed ranges, e.g., A, B and C are such that

$$\begin{aligned} A &= \left\{[a_{ij}] : a_{ij}^m \le a_{ij} \le a_{ij}^M,\ i, j = 1, 2, \ldots, n\right\}, \\ B &= \left\{[b_{ij}] : b_{ij}^m \le b_{ij} \le b_{ij}^M,\ i, j = 1, 2, \ldots, n\right\}, \\ C &= \left\{[c_{ij}] : c_{ij}^m \le c_{ij} \le c_{ij}^M,\ i, j = 1, 2, \ldots, n\right\}. \end{aligned} \tag{2}$$

τ is the positive time delay, and $\phi(\cdot)$ is the given continuously differentiable function on $[-h, 0]$.

In the past, a number of reports have proposed the stability analysis of interval systems [6–8]. Park [8] presented a delay-independent criterion for asymptotic stability of the system given in (1). The derived sufficient conditions are expressed in terms of the spectral radius of the matrix, but there is a technical error in the proof of the main theorem (see Section 2).

The work is organized as follows. We first point out the error in [8], in Section 2. Then, to reduce the conservatism, we provide explicit conditions in terms of the structured singular value, not the spectral radius of the matrix, in Section 3. Finally, we give an example to demonstrate the conditions, in Section 4.

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2. Problem statement

The following notation defined in Park [8] is employed in this work:

$$\begin{aligned} &A^m = (a_{ij}^m), \quad A^M = (a_{ij}^M); \quad B^m = (b_{ij}^m), \\ &B^M = (b_{ij}^M); \quad C^m = (c_{ij}^m), \quad C^M = (c_{ij}^M); \end{aligned} \tag{3}$$

$$\begin{aligned} &A_0 = (a_{ij}^0) = \frac{1}{2}(a_{ij}^m + a_{ij}^M) = \frac{1}{2}(A^m + A^M), \\ &B_0 = (b_{ij}^0) = \frac{1}{2}(b_{ij}^m + b_{ij}^M) = \frac{1}{2}(B^m + B^M), \\ &C_0 = (c_{ij}^0) = \frac{1}{2}(c_{ij}^m + c_{ij}^M) = \frac{1}{2}(C^m + C^M); \end{aligned} \tag{4}$$

$$\begin{aligned} &A_\delta = (a_{ij} - a_{ij}^0) = A - A_0, \quad B_\delta = (b_{ij} - b_{ij}^0) = B - B_0, \quad C_\delta = (c_{ij} - c_{ij}^0) = C - C_0; \\ &A' = a_{ij}^M - a_{ij}^0 = A^M - A^0, \quad B' = b_{ij}^M - b_{ij}^0 = B^M - B^0, \quad C' = c_{ij}^M - c_{ij}^0 = C^M - C^0; \\ &|A_\delta| \le A', \quad |B_\delta| \le B', \quad |C_\delta| \le C'. \end{aligned} \tag{5}$$

On the basis of this notation, Park derived the following theorem in his paper [8].

Theorem 2.1 ([8]). *The interval neutral system given in (1) is asymptotically stable if the following inequalities are satisfied:*

$$\begin{aligned} &(1)\ \|C_0\| + \|C'\| < 1, \\ &(2)\ \rho[F_M\{A' + |B_0| + B' + \frac{1}{1 - (\|C_0\| + \|C'\|)}(|C_0A_0| + |C_0|A' + |C_0B_0| \\ &\quad + |C_0|B' + C'(|A_0| + A' + |B_0| + B'))\}] < 1, \end{aligned} \tag{6}$$

where F_M denotes a matrix formed by taking the maximum magnitude of each element of $F(s) = (sI - A)^{-1}$ for $\mathscr{R}e(s) > 0$.

In the proof of the Theorem 2.1, the author used the inequality

$$\rho(|R||T|) \le \rho(\|R\||T|). \tag{7}$$

It is easy to see that, in general, the inequality (7) does not hold for given matrices R and T. For example,

$$R = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}, \qquad T = \begin{bmatrix} -4 & -8 \\ 0 & 5 \end{bmatrix},$$

and then

$$|R| = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}, \qquad |T| = \begin{bmatrix} 4 & 8 \\ 0 & 5 \end{bmatrix}, \qquad |R||T| = \begin{bmatrix} 4 & 8 \\ 4 & 18 \end{bmatrix}, \quad \rho(|R||T|) = 20.$$

But

$$\|R\| = 2.2882, \qquad \|R\| \cdot |T| = \begin{bmatrix} 9.1528 & 18.3056 \\ 0 & 11.4110 \end{bmatrix}, \qquad \rho(\|R\||T|) = 11.4410,$$

so $|R| \cdot |T| < \|R\| \cdot |T|$ does not hold and $\rho(|R||T|) > \rho(\|R\||T|)$, which is in contradiction with the inequality (7).

Therefore, the following inequality:

$$\begin{aligned} &\rho\left[|F(s)|\left\{A' + |B_0| + B' + |(I - T)^{-1}|(|TA_0 + TA' + \xi TB_0 + \xi TB'|)\right\}\right] \\ &\quad \le \rho\left[F_M\left\{A' + |B_0| + B' + \|(I - T)^{-1}\|(|TA_0| + |TA'| + |\xi TB_0| + |\xi TB'|)\right\}\right], \end{aligned}$$

is not feasible. Thus, the result (6) in Theorem 2.1 does not hold.

3. Main results

In this section, we first correct Theorem 2.1, then give a new stability condition for the system (1), which is based on the matrix structured singular value. Throughout this work, we will always assume that matrix A is a Hurwitz matrix.

Lemma 3.1 ([9]). *If $a_D = \sup\{\mathscr{R}e(s) : \Delta(s) = 0\}$ and $a_D < 0$, then the neutral system (1) is asymptotically stable.*

Lemma 3.2 (*[10]*). *Consider any $n \times n$ matrices R, T and V, with $|R| \le V$. Then:*

(a) $|RT| \le |R||T| \le V|T|$,
(b) $|R+T| \le |R|+|T| \le V+|T|$,
(c) $\rho[R] \le \rho[|R|] \le \rho[V]$,
(d) $\rho[RT] \le \rho[|R||T|] \le \rho[V|T|]$,
(e) $\rho[R+T] \le \rho[|R+T|] \le \rho[|R|+|T|] \le \rho[V+|T|]$.

Lemma 3.3 (*[11]*). *Consider any $n \times n$ matrices R. If $\rho(R) < 1$, then $\det(I \pm R) \ne 0$ and*

$$(I-R)^{-1} = I + R + R^2 + \cdots. \tag{8}$$

If $\|R\| < 1$, then $(I-R)^{-1}$ exists, and $\|(I-R)^{-1}\| \le 1/(1-\|R\|)$.

Theorem 2.1 can be corrected as follows.

Theorem 3.1. *The interval neutral system given in (1) is asymptotically stable if the following inequalities are satisfied:*

$$\begin{aligned}
&(1)\ \rho(|C_0|+C') < 1,\\
&(2)\ \rho[F_M\{A'+|B_0|+B'+(I-(|C_0|+C'))^{-1}(|C_0A_0|+|C_0|A'+|C_0B_0|\\
&\qquad +|C_0|B'+C'(|A_0|+A'+|B_0|+B'))\}] < 1,
\end{aligned} \tag{9}$$

where F_M denotes a matrix formed by taking the maximum magnitude of each element of $F(s) = (sI-A)^{-1}$ for $\mathcal{R}e(s) > 0$.

Proof. According to the paper [8], the system (1) is asymptotically stable if the matrix $(I - C\exp(-\tau s))^{-1}$ exists and

$$\rho[F(s)\{A_\delta + \xi B_0 + \xi B_\delta + (I-T)^{-1}T(A_0 + A_\delta + \xi B_0 + \xi B_\delta)\}] < 1$$

for $\mathcal{R}e(s) > 0$, where $\xi = \exp(-\tau s)$, $T = \xi C$.

According to Lemma 3.3, if the condition (1) of Theorem 3.1 holds, then the matrix $(I - C\exp(-\tau s))^{-1}$ exists.

Using Lemma 3.2, for $\mathcal{R}e(s) > 0$, we have

$$|TA_0| = |\xi(C_0 + C')A_0| \le |\xi C_0 A_0| + |\xi C' A_0| \le |C_0A_0| + C'|A_0|. \tag{10}$$

Similarly,

$$|TA_\delta| \le |C_0|A' + C'A', \qquad |\xi TB_0| \le |C_0B_0| + C'|B_0|, \qquad |\xi TB_\delta| \le |C_0|B' + C'B'. \tag{11}$$

Now, using Lemma 3.2 and (10) and (11), we obtain

$$\begin{aligned}
&\rho[F(s)\{A_\delta + \xi B_0 + \xi B_\delta + (I-T)^{-1}T(A_0 + A_\delta + \xi B_0 + \xi B_\delta)\}]\\
&\quad \le \rho[|F(s)|\{|A_\delta| + |\xi B_0| + |\xi B_\delta| + |(I-T)^{-1}(TA_0 + TA_\delta + \xi TB_0 + \xi TB_\delta)|\}]\\
&\quad \le \rho[F_M\{A' + |B_0| + B' + |(I-T)^{-1}| \cdot |TA_0 + TA_\delta + \xi TB_0 + \xi TB_\delta|\}]\\
&\quad \le \rho[F_M\{A' + |B_0| + B' + |I + T + T^2 + \cdots| \cdot (|TA_0| + |TA_\delta| + |\xi TB_0| + |\xi TB_\delta)|\}]\\
&\quad \le \rho[F_M\{A' + |B_0| + B' + (|I| + |T| + |T|^2 + \cdots)(|C_0A_0| + |C_0|A' + C'A_0 + C'A'\\
&\qquad + |C_0B| + C'|B| + |C_0||B'| + C'B')\}]\\
&\quad \le \rho[F_M\{A' + |B_0| + B' + (I - |T|)^{-1}(|C_0A_0| + |C_0|A' + C'A_0 + C'A' + |C_0B| + C'|B| + |C_0||B'| + C'B')\}]\\
&\quad \le \rho[F_M\{A' + |B_0| + B' + (I - (|C_0| + C'))^{-1}(|C_0A_0| + |C_0|A' + |C_0B_0| + |C_0|B' + C'(|A_0| + A' + |B_0| + B'))\}]\\
&\quad \le 1.
\end{aligned} \tag{12}$$

Thus, the proof is completed. □

Next, a new condition, in terms of the structured singular value, is provided to ensure the stability of the interval system. Before we develop our main results, we state some useful definitions and lemmas.

Definition 3.1 (*[12]*). Suppose that M is a complex partitioned as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \tag{13}$$

and Δ is a matrix with an appropriate dimension; then the upper and lower linear fractional transformations (LFTs) are defined, respectively, by

$$\begin{aligned}
F_u(M, \Delta) &:= M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12},\\
F_l(M, \Delta) &:= M_{11} + M_{12}\Delta(I - M_{22}\Delta)^{-1}M_{21}.
\end{aligned} \tag{14}$$

Definition 3.2 ([13]). For a complex matrix $M \in \mathcal{C}^{n\times n}$, the structured singular value of M with respect to a block structure set Δ is defined by

$$\mu_\Delta(M) := \frac{1}{\min\limits_{\tilde{\Delta}\in\Delta}\left\{\bar{\sigma}(\tilde{\Delta}) : \det(I - M\tilde{\Delta}) = 0\right\}}.$$

If there is no $\tilde{\Delta} \in \Delta$ such that $\det(I - M\tilde{\Delta}) = 0$, then $\mu_\Delta(M) := 0$.

Now, consider the interval system (1) with the center matrix $A_0 := (A^m + A^M)/2$ and the difference matrix $\tilde{A} := (A^M - A^m)/2 =: [\tilde{a}_{ij}]$. Let e_i denote a standard basis column vector in $\mathcal{R}^n$ with all entries 0 except for the ith component which is 1. With the help of e_i, we can define an $n \times n$ constant matrix $A_i := e_i e_i^T \tilde{A},\ i = 1, 2, \ldots, n$. Namely, the matrix A_i contains the ith row of $\tilde{A}$, and the other entries are 0. Using Definition 3.1, an LFT representation of an interval matrix may be constructed as in the following lemma.

Lemma 3.4. *The interval matrix A as in (1) can be expressed as*

$$A = A_0 + [A_1, A_2, \ldots, A_n]\begin{bmatrix}\lambda_1^{(A)}\\ \lambda_2^{(A)}\\ \vdots\\ \lambda_n^{(A)}\end{bmatrix}, \tag{15}$$

where $\lambda_i^{(A)} = \mathrm{diag}[\lambda_{i1}^{(A)}, \ldots, \lambda_{in}^{(A)}]$, $-1 \le \lambda_{ij}^{(A)} \le 1$ $(1 \le i, j \le n)$.

Proof. From Ref. [14], we have that the interval matrix in (1) can be expressed as

$$A = F_l(M, \Delta_A) := F_l\left(\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}, \ \Delta_A\right), \tag{16}$$

where $M_{11} = A_0$, $M_{12} = [A_1, A_2, \ldots, A_n]$, $M_{21} = [I_n, I_n, \ldots, I_n]^T$, $M_{22} = 0_{n^2}$, and $\Delta_A = \mathrm{diag}[\lambda_{11}^{(A)}, \ldots, \lambda_{1n}^{(A)}, \lambda_{21}^{(A)}, \ldots, \lambda_{2n}^{(A)}, \ldots, \lambda_{n1}^{(A)}, \ldots, \lambda_{nn}^{(A)}]$ $(-1 \le \lambda_{ij} \le 1,\ 1 \le i, j \le n)$, such that

$$A = A_0 + [A_1, A_2, \ldots, A_n]\Delta_A\begin{bmatrix}I_n\\ I_n\\ \vdots\\ I_n\end{bmatrix} = A_0 + [A_1, A_2, \ldots, A_n]\begin{bmatrix}\lambda_1^{(A)}\\ \lambda_2^{(A)}\\ \vdots\\ \lambda_n^{(A)}\end{bmatrix}.$$

Thus, the proof is completed. □

Similarly, for matrices B and C, we also have that

$$B = B_0 + [B_1, B_2, \ldots, B_n]\begin{bmatrix}\lambda_1^{(B)}\\ \lambda_2^{(B)}\\ \vdots\\ \lambda_n^{(B)}\end{bmatrix}, \qquad C = C_0 + [C_1, C_2, \ldots, C_n]\begin{bmatrix}\lambda_1^{(C)}\\ \lambda_2^{(C)}\\ \vdots\\ \lambda_n^{(C)}\end{bmatrix}, \tag{17}$$

where

$$\lambda_i^{(B)} = \mathrm{diag}[\lambda_{i1}^{(B)}, \ldots, \lambda_{in}^{(B)}], \qquad \lambda_i^{(B)} = \mathrm{diag}[\lambda_{i1}^{(C)}, \ldots, \lambda_{in}^{(C)}] \quad (i = 1, 2, \ldots, n).$$

Next, define

$$A^\Delta = [A_1, A_2, \ldots, A_n]\begin{bmatrix}\lambda_1^{(A)}\\ \lambda_2^{(A)}\\ \vdots\\ \lambda_n^{(A)}\end{bmatrix}, \qquad B^\Delta = [B_1, B_2, \ldots, B_n]\begin{bmatrix}\lambda_1^{(B)}\\ \lambda_2^{(B)}\\ \vdots\\ \lambda_n^{(B)}\end{bmatrix}, \tag{18}$$

and $C^\Delta = [C_1, C_2, \ldots, C_n]\begin{bmatrix}\lambda_1^{(C)}\\ \lambda_2^{(C)}\\ \vdots\\ \lambda_n^{(C)}\end{bmatrix}$; then $|A^\Delta| \le [|A_1|, |A_2|, \ldots, |A_n|]\begin{bmatrix}|\lambda_1^{(A)}|\\ |\lambda_2^{(A)}|\\ \vdots\\ |\lambda_n^{(A)}|\end{bmatrix} \le \sum_{i=1}^n |A_i| = A'$.

Similarly, we have that

$$|B^{\Delta}| \leq \sum_{i=1}^{n} |B_i| = B', \qquad |C^{\Delta}| \leq \sum_{i=1}^{n} |C_i| = C'. \tag{19}$$

On the basis of the above definitions and lemmas, we obtain the following theorem.

Theorem 3.2. *The interval neutral delay-dependent system given in (1) is asymptotically stable if the following inequalities are satisfied:*

$$\begin{aligned} &(1)\ \rho(|C_0| + C') < 1, \\ &(2)\ \mu_{\Delta}(M) < 1, \quad M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \end{aligned} \tag{20}$$

where

$$\begin{aligned} &M_{11} = F_M(A' + B' + |B_0|), \\ &M_{12} = F_M(|C_0| + C')(I - (|C_0| + C'))^{-1}[(|A_0|, |B_0|, |A_1|, \ldots, |A_n|, |B_1|, \ldots, |B_n|]), \\ &M_{21} = [I_n, \ldots, I_n], \\ &M_{22} = 0_{6n}. \\ &\Delta = \operatorname{diag}(I_{3n}, |\lambda_1{}^{(A)}|, \ldots, |\lambda_n{}^{(A)}|, |\lambda_1{}^{(B)}|, \ldots, |\lambda_n{}^{(B)}|) \end{aligned} \tag{21}$$

and F_M is defined as in Theorem 2.1.

Proof. The characteristic equation of the system given in (1) is

$$\lambda(s) = \det[sI - A - (B + sC)e^{-\tau s}] = 0.$$

Since $\det(RT) = \det(R)\det(T)$ for any two $n \times n$ matrices R and T, we have

$$\lambda(s) = \det[I - Ce^{-s\tau}] \det\left[sI - (I - Ce^{-s\tau})^{-1}(A + Be^{-s\tau})\right].$$

From (20) and Ref. [14], we have

$$\mu_{\Delta}(M) < 1 \Leftrightarrow \rho(F_l(M,\ \Delta)) < 1, \quad \sigma(\Delta) \leq 1. \tag{22}$$

According to Lemma 3.1, the system (1) is asymptotically stable if $\lambda(s) \neq 0$ for $\mathcal{R}e(s) \geq 0$. Therefore, if we can show that

$$\det[sI - (I - Ce^{-\tau s})^{-1}(A + Be^{-\tau s})] \neq 0, \quad \text{for } \mathcal{R}e(s) \geq 0, \tag{23}$$

then system (1) is asymptotically stable.

Let us define $T = Ce^{-s\tau}$ using Lemma 3.1 and the inequality $(I - T)^{-1} = I + (I - T)^{-1}T$; then, (23) becomes

$$\begin{aligned} &\det[sI - (I - T)^{-1}(A + Be^{-s\tau})] \neq 0 \\ &\Leftrightarrow \det[sI - (I + T(I - T)^{-1})(A + Be^{-s\tau})] \neq 0 \\ &\Leftrightarrow \det[(sI - A_0) - A^{\Delta} - (B_0 + B^{\Delta})e^{-s\tau} - T(I - T)^{-1}(A_0 + A^{\Delta} + (B_0 + B^{\Delta})e^{-s\tau})] \neq 0 \\ &\Leftrightarrow \det[sI - A_0] \det[I - (sI - A_0)^{-1}\{A^{\Delta} + (B_0 + B^{\Delta})e^{-s\tau} \\ &\qquad + (C_0 + C^{\Delta})e^{-s\tau}(I - T)^{-1}(A_0 + A^{\Delta} + (B_0 + B^{\Delta})e^{-s\tau})\}] \neq 0 \\ &\Leftrightarrow \det[sI - A_0] \det[I - F(s)\{A^{\Delta} + (B_0 + B^{\Delta})e^{-s\tau} + (C_0 + C^{\Delta})e^{-s\tau}(I - T)^{-1}(A_0 + A^{\Delta} + (B_0 + B^{\Delta})e^{-s\tau})\}] \neq 0. \end{aligned} \tag{24}$$

Since A_0 is a Hurwitz matrix, $\det[sI - A] \neq 0$ for $\mathcal{R}e(s) \geq 0$. So, Eq. (24) is further simplified to

$$\det[I - F(s)\{A^{\Delta} + (B_0 + B^{\Delta})e^{-s\tau} + (C_0 + C^{\Delta})e^{-s\tau}(I - T)^{-1}(A_0 + A^{\Delta} + (B_0 + B^{\Delta})e^{-s\tau})\}] \neq 0. \tag{25}$$

Thus, if we can show that

$$\rho[F(s)\{A^{\Delta} + (B_0 + B^{\Delta})e^{-s\tau} + (C_0 + C^{\Delta})e^{-s\tau}(I - T)^{-1}(A_0 + A^{\Delta} + (B_0 + B^{\Delta})e^{-s\tau})\}] \leq 1, \tag{26}$$

for $\mathcal{R}e(s) \geq 0$, then Eq. (23) is satisfied, by Lemma 3.4. Moreover, using (18), we can get

$$\begin{aligned} |C| &\leq |C_0| + [|C_1|,\ |C_2|, \ldots,\ |C_n|][|(\lambda_1^C)^T|,\ |(\lambda_2^C)^T|, \ldots, |(\lambda_n^C)^T|]^T \\ &\leq \sum_{i=0}^{n} |C_i|. \end{aligned} \tag{27}$$

Thus, from Lemma 3.4, we have for $\mathcal{R}e(s) \geq 0$,

$$|(I - Ce^{-s\tau})^{-1}| = |I + Ce^{-s\tau} + C^2 e^{-2s\tau} + \cdots| \leq |I| + |C| + |C|^2 + \cdots \leq \left(I - \sum_{i=1}^{n} |C_i|\right)^{-1}. \tag{28}$$

Now, using Lemma 3.3 together with Lemma 3.4 and (26)–(28), we can obtain for $\mathcal{R}e(s) \geq 0$ that

$$\begin{aligned}
&\rho[F(s)\{A^{\Delta} + (B_0 + B^{\Delta})e^{-s\tau} + (C_0 + C^{\Delta})e^{-s\tau}(I - T)^{-1}(A_0 + A^{\Delta} + (B_0 + B^{\Delta})e^{-s\tau})\}] \\
&\quad \leq \rho[|F(s)|\{|A^{\Delta}| + |(B_0 + B^{\Delta})e^{-s\tau}| + |(C_0 + C^{\Delta})e^{-s\tau}\|(I - T)^{-1}\|(A_0 + A^{\Delta} + (B_0 + B^{\Delta})e^{-s\tau})|\}] \\
&\quad \leq \rho\left[F_M\left\{\sum_{i=1}^{n}|A_i| + \sum_{i=0}^{n}|B_i| + B_0 + \left(|C_0| + \sum_{i=0}^{n}|C_i|\right)\left(I - \left(C_0 + \sum_{i=1}^{n}|C_i|\right)\right)^{-1}\left(\sum_{i=0}^{n}(|A_i| + |B_i|)\right)\right\}\right] \\
&\quad = \rho[F_M\{A' + B' + |B_0| + (|C_0| + C')(I - (|C_0| + C'))^{-1} \\
&\qquad \times ([|A_0|, |B_0|, |A_1|, \ldots, |A_n|, |B_1|, \ldots, |B_n|]\Delta[I_n, I_n, \ldots, I_n]^T)\}] \\
&\quad = \rho[F_I(M, \Delta)].
\end{aligned} \tag{29}$$

From (20) and (21), we have that

$$\rho[F_I(M, \Delta)] < 1.$$

Thus, the proof is completed. □

4. Illustrative examples

To demonstrate the validity of our criteria, let us examine the following simple example:

Example 4.1. Consider the interval neutral differential system described by (1), where

$$A^m = \begin{bmatrix} -7 & 0.5 \\ 0 & -5 \end{bmatrix}, \quad A^M = \begin{bmatrix} -5 & 1.5 \\ 0 & -3 \end{bmatrix}, \quad B^m = \begin{bmatrix} -0.4 & 0.2 \\ 0 & -0.3 \end{bmatrix},$$

$$B^M = \begin{bmatrix} -0.2 & v \\ 0.2 & 0.3 \end{bmatrix}, \quad C^m = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad C^M = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix},$$

where v is a scalar parameter, for which we shall find, by Theorem 3.1, the upper and lower bounds that guarantee the stability of the system (1).

We first have that

$$A_0 = \begin{bmatrix} -6 & 1 \\ 0 & -4 \end{bmatrix}, \quad B_0 = \begin{bmatrix} -0.3 & 0.1 + 0.5v \\ 0.1 & 0 \end{bmatrix}, \quad C_0 = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.1 \end{bmatrix}; \tag{30}$$

$$A' = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}, \quad B' = \begin{bmatrix} 0.1 & 0.5v - 0.1 \\ 0.1 & 0.3 \end{bmatrix}, \quad C' = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.1 \end{bmatrix}. \tag{31}$$

The function matrices $F(s)$ are computed as

$$F(s) = \begin{bmatrix} \dfrac{1}{s+6} & \dfrac{1}{(s+4)(s+6)} \\ 0 & \dfrac{1}{s+4} \end{bmatrix}, \quad F_M = \begin{bmatrix} \dfrac{1}{6} & \dfrac{1}{24} \\ 0 & \dfrac{1}{4} \end{bmatrix}. \tag{32}$$

Then, from (9), we have that the bound of v for guaranteeing the asymptotic stability of the system (1) is

$$0.2 \leq v < 1.7.$$

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